A PRACTICAL TREATISE

ON

RAILWAY CURVES

AND

LOCATION,

FOR YOUNG ENGINEERS.

CONTAINING A FULL DESCRIPTION OF THE INSTRUMENTS, THE MANNER OF ADJUSTING THEM, AND THE METHODS OF PROCEEDING IN THE FIELD,—NEW AND SIMPLE FORMULÆ FOR COMPOUND AND REVERSE CURVING,—RULES FOR CALCULATING EXCAVATION AND EMBANKMENT,—STAKING OUT WORK, &c., TOGETHER WITH TABLES OF NATURAL SINES AND TANGENTS, RADII, CHORDS, ORDINATES, AND OTHERS OF GENERAL USE IN THE PROFESSION.

BY

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PREFACE.

THE located line for railway is a series of curves and straight lines, or tangents. These are first plotted to a large scale from data gathered on preliminary survey. It is therefore desirable that all explorations should be made with extreme care, as upon their correctness depend, in no small degree, the labour and time required in location. It were better for accuracy that all angles should be made and recorded from the plates, and the needle used only as a test, or check. Good chaining is indispensable. Great attention, too, should be given to the proper use of the slope instrument. By these means a working map can be constructed in the office upon which the proposed location, grade lines, &c., may be traced with tolerable resemblance to fact. Still many errors attach to both data and map, and these, together with the unexpected obstacles encountered in the field, require ready knowledge of the means for overcoming them.

It has been my design to present this knowledge to my younger fellows in the profession. I have endeavoured to

do it lucidly and concisely—without supposing unusual cases—without prolix proof or complex figuring. The problems given are of frequent occurrence, and the tables appended will be found useful and correct.

To Strickland Kneass, a gentleman whose professional abilities are well known, I return thanks for valuable assistance. I would likewise make my acknowledgments, for useful suggestions, to Charles Delisle, an engineer of high mathematical attainment.

I am aware that much more might have been said—much more suggested—on the subject of location; but a field book being the object, the compact plan precluded any extensive essay.

If the work with brevity combines clearness, and is comprehensive withal, it is the work intended.

W. F. Shunk.

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EXPLANATIONS.

All railway curves are parts of circles. They are designated *generally* from their character as simple, compound, or reverse; and *specifically* from the central angle subtended by a chord of 100 feet at the circumference, this being the length of the chain in common use. It is found that the circle described with radius of 5730 feet has a circumference of 36,000 feet. Since there are 360° in the circle, the central angle subtended by a chord of 100 feet is, in this case, equal to 1°, and the curve is named a *one degree* curve. So likewise in a circle with radius of 2865 feet, half of 5730, the central angle corresponding to the chord 100 is 2°; the curve is then called a *two degree* curve.

The beginning of a curve is called the point of curvature, or simply the P. C., and its termination the point of tangent, marked P. T.

A compound curve is composed of two curves of different radii, turning in the same direction, and having a common tangent at their point of meeting. This point is called the point of compound curvature, or P. C. C.

A reverse curve is composed of two curves turning in different directions, and having a common tangent at their point of meeting, which latter is named the point of reversed curvature, or the P. R. C.

All sines and tangents made use of in this work are from the table at the end of the volume. For calculating curves it is not necessary to use more than four decimals.

A Bench is a shoulder hewn with the axe on the buttressed base of a tree, and so shaped at the top as to afford footing to the rod. The tree is blazed and the elevation of the bench marked on it with red chalk. Benches serve as permanent reference points to the level. They are placed, where it is possible, about one thousand feet apart.

Points. The operation termed pointing is the fact of putting a peg firmly into the ground, and of driving in its top a tack, or making thereon an indentation whose place is indicated by cross keel marks, directly in the line of collimation of the transit. Thus true lines are traced on the ground, and angles measured accurately. When the transit is set over a point it is so posited that the plumb hangs immediately above the tack head. If the head plate of the tripod be much inclined the plumb should be examined after levelling the instrument, as that operation disturbs it to some extent.

Stations. The line of a survey is marked on the ground at regular intervals, by stakes two feet in length, blazed, and numbered from 0 up in arithmetical progression. These stakes are named stations. On exploration they are commonly placed two hundred, and on location, one hundred feet apart.

It is customary, when locating, to drive pegs even with the surface along the true line, and to place the stations a couple of feet to the right, numbers facing in, to show their

position. The pegs are less liable to disturbance from frost, animals, &c.

In locating for construction stakes are driven on sharp curves at intervals of 50, sometimes 25 feet.

The Chain in general use for railway surveys is made of soft iron. It is 100 feet long, and divided into 100 links, each one foot in length. At every tenth link is attached a brass drop, toothed so as to indicate its distance from the end. It presents the advantages of durability, accuracy, and expedition.

A PRACTICAL TREATISE

ON

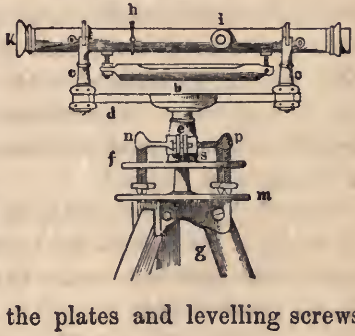
RAILWAY CURVES AND LOCATION.

ARTICLE I.

OF THE INSTRUMENTS.

THE LEVEL.

THE level is an instrument used in ascertaining the undulations of the ground along the line of a survey, and of measuring these irregularities accurately in reference to an assumed base called the datum. It consists mainly of the telescope *k i*, the spirit-level and its encasement *b*, the Y's *c c*, the rectangular bar *o d*, the axis *e*, the plates and levelling screws *f m*, and the tripod *g*.



In the tube of the telescope at *h*, and at right angles to its axis, is placed a flat ring, called the diaphragm. To this ring the cross-hairs are attached—two delicate spider lines stretched over it vertically and horizontally, and intersecting at the centre. It is held in position by four

slightly movable screws, which pierce the tube in the direction of the "cross-hairs." *i* is a milled head for adjusting the focus of the object glass, and *k* an inserted tube, containing several lenses, which may be moved out or in so as to make the spider-lines distinctly visible.

A straight line looked along from the eye glass at *k* through the intersection of the cross-hairs is the *line of sight*, technically named the *line of collimation.*

The immediate supports of the telescope are called the Y's, from their resemblance to that letter. If a small arch were sprung between the two legs of the Y it would give a good idea of the clasping pieces which hold the telescope in place. They are jointed to one leg and secured to the other by pins which may be withdrawn and the pieces turned back in order to remove the telescope, or change it end for end.

The Y's are attached at right angles to the bar *d*, which, again, is connected firmly at right angles with the hollow axis *e*. This latter fits closely over and is revolvable horizontally around a *solid* axis *s*, which, passing through the plate *f*, is secured to the head of the tripod by means of a loose ball-and-socket joint. The plate *f* has four levelling screws inserted in it; with these the instrument may be brought to a horizontal position even when the lower plate is considerably inclined.

One of the Y's is movable for a short space up or down by means of the capstan-head screw *o*. The spirit-level is likewise movable both vertically and laterally by means of screws at either end.

n is a clamp screw, and *p* a tangent-screw for slightly turning the telescope in a horizontal direction.

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To adjust the Level.

First. To make the line of collimation coincide with the axis of the telescope.

Set the instrument firmly, and direct the telescope toward some distant, distinct object, such as a nail-head. Clamp fast, and with tangent-screw fix the line of collimation upon the object accurately. Revolve the telescope half way round in the Y's, *i. e.* until the bubble is above it, and if the horizontal spider-line still covers the point, it requires no adjustment. If it does not, reduce the error one-half by means of the diaphragm screws, and complete the reduction with the capstan-head screw. Revolve the telescope round to its first position, and if the horizontal line and point do not then coincide, repeat the operation until they do, in any position of the telescope. In similar wise the vertical hair may be adjusted, when the line of collimation should cover the point through an entire revolution of the telescope.

Great care should be taken in this as well as in all other adjustments of cross-hairs, that the opposite screw of the diaphragm be loosened before tightening its fellow, or injury to the instrument must result.

Second. To make the axis of the spirit-level parallel to the line of collimation.

With levelling screws bring the bubble to the middle of its tube, reverse the telescope in its Y's, and if the bubble does not then stand in the middle correct one-half the deviation with the screw at the left end of the bubble-case, and the other half with the capstan-head screw. Again reverse the telescope in its Y's, and, if necessary, repeat the operation.

Now revolve the telescope a short distance in its Y's, so as to bring the spirit-level to one side of its lowest position. If the bubble deviates from the middle, correct the error

with the lateral screws at the right end of the bubble-case, and examine the previous adjustment before lifting the instrument.

Third. To bring the line of collimation parallel to the bar.

Turn the telescope until it stands directly over two of the levelling screws, and with them bring the bubble to the middle of the tube. Then revolve the telescope horizontally until it stands over the same screws, changed end for end. If the bubble does not still stand in the middle of the tube, correct one-half the deviation with the capstan-head, and one-half with the levelling screws.

Place the telescope over the other levelling screws and proceed in a similar manner, and continue the corrections until the bubble stands without varying during an entire revolution of the instrument upon its axis.

This completes the adjustment of the level.

THE ROD.

The rod used in levelling consists of a staff and a target, which latter is so attached to the staff as to be movable along it from end to end. The rod is commonly seven feet long, but, being composed of two rectangular pieces fitted together by means of a sliding groove, it can be extended to nearly double that length. It is graduated to feet and tenths of a foot. The target is a circle of wood or iron, usually four-tenths in diameter, and divided into quadrantal sectors by a horizontal and vertical line which intersect at its centre. The sectors are painted alternately red and white, so that their dividing lines are visible at a considerable distance. On the back of the target, where it meets the graduated side of the rod, is fixed a chamfered brass edging, whereon the space of one-tenth is graven from the centre down. This is subdivided into ten spaces marking

hundredths, and these latter divided into halves, so that the height of the middle of the target above the base of the rod may be accurately read to within ·005 of a foot.

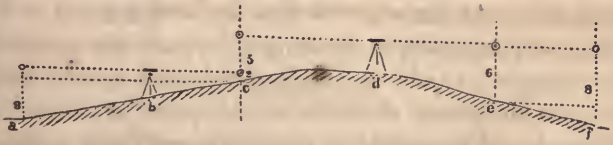
There is a similar graduated tenth on the standing part of the rod, to be used for high sights when the sliding groove comes into play.

Both target and rod are provided with clamp screws.

LEVELLING.

The operation technically called levelling is performed thus:—

Suppose *a* the starting point, or zero, in reference to which all the inequalities of the surface along the line of survey are measured, as at the points *c*, *e*, *f*. The horizontal line *a f* is called the *datum* line. This is arbitrarily



assumed. It may be considered, for example, at any distance *above* the point *a*, and the irregularities of the ground measured from an imaginary level line in ether; but for convenience of figuring, and other politic reasons, it is customary in seaport towns to take high tide as datum. Inland, the summer surface of the nearest stream, or, when commencing on a ridge, the highest neighbouring knoll is assumed.

Well! suppose *a* to be zero, and the instrument, for instance, set and levelled at *b*. Stand the rod at *a*, and slide the target up until its cross-lines are covered by the cross-hairs in the telescope; *i. e.*, until the line of collimation coincides with the centre of the target. The leveller directs the movements of the target by raising or lowering

his hand. A circular motion of the hand signifies "make fast." The bubble should always be examined before the rod is taken down, and the latter should be read twice, or, if convenient, shown to the leveller, in order to guard against mistake. If in this case it reads 8 feet, the height of the instrument is then 8 feet above *a*. To find the elevation of *c* above *a*, take the rod thither and lower the target until coincidence results as before. If the rod reads 2 feet, of course *c* is 8 — 2 = 6 feet above *a*.

If it is necessary to lift the instrument here, a small peg is driven at *c* before sighting to that point, to insure firm footing for the rod. Sighting back from the new position, *d*, the rod reads 5 feet; then 5 + 6, the elevation of *c* above *a*, = 11, the height of the telescope at *d* above *a*. If at *e* the reading is 6, the elevation of that point is 11 — 6 = 5, and if at *f* the reading is 8 the elevation of that point in reference to *a* is 11 — 8 = 3, marked + 3.

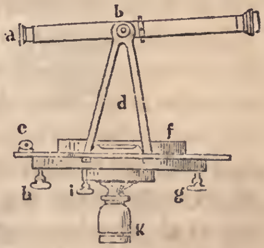
The rule, therefore, in levelling is, at each new stand of the instrument, to add the reading of the rod sighted back at, to the discovered elevation of the point at which the rod stands, for the height of the instrument; and to *subtract* from this height the reading of the rod at any points observed from the new position in order to find the elevation of those points. The above is noted in the field-book as follows:

Station.	Rod.	Height of Instrument.	Total, or Elevation.
a	8·00	8·00	00
c	2·00		+ 6.00
	5·00	11·00	
e	6·00		+ 5·00
f	8·00		+ 3·00

The advantages of this method of levelling over the old system of backsights and foresights are, that it affords readier facilities for testing the correctness of the work, and it may be carried on more rapidly. By the old plan each sight at the rod was linked with that which preceded it, and added one more to a continuous calculation in which a single error affected all the following work. Here, however, if haste is required, the calculation of the intermediate sights or "cuttings" may be omitted entirely while in the field, the reading of the rod only being set down; the "totals" may be worked from peg to peg, and the liabilities to mistake thus decreased about eighty per cent.

OF THE TRANSIT.

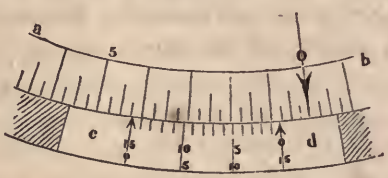
The transit is an instrument for measuring horizontal angles. It consists of the telescope *a c*, the Y's *d*, the compass-box, &c., *e g*, and the axis *k*. The telescope is furnished like that of the level, and the instrument is similarly fitted to its tripod. The telescope revolves in a vertical circle, and is attached to the Y's by means of a transverse axis whose extremities turn in smooth journals at the head of the Y's. The body of the instrument at *f* contains a magnetic needle, with its usual circular surrounding, graduated to degrees and quarter degrees. The flooring of this box has, on one side, an opening with chamfered edge upon which the vernier is engraved. This latter, together with the telescope, Y's, and all the upper part of the instrument, is made to revolve by means of the screw *h*, upon a solid plate beneath, which is likewise graduated from 0 to 180° each way. Thus angles may be measured



accurately without using the needle at all. It need be regarded merely as a check. *g* is a clamp screw for securing the plates together, and *i* a screw for fastening the needle so as to prevent its vibrations while the instrument is being carried from place to place. A plumb is suspended from the axis of the transit, by means of which its centre may be placed over a point on the ground.

THE VERNIER.

The vernier, in the transit, is a graduated index which serves to subdivide the divisions of the graduated arc on the lower plate. There are many varieties of the vernier, but familiarity with one renders easy the acquaintance with all, since the same general principle is pervading.



The figure represents a common form. Let *a b* be part of any graduated arc, and *c d* the vernier. It will be observed that the degrees on the limb are divided into spaces of 15′ each. Now if the vernier be made equal in length to fourteen of those spaces, and be further divided into *fifteen equal parts*, it is evident that each of these parts will contain 14′.

Then, if 0 of the vernier coincides with any division of the limb, the first line of the vernier to the left will be just one minute behind the first line of the limb to the left; the *second* vernier line *two* minutes behind the second limb line, and so on; so that if the vernier be moved to the left over the space of 15′ on the limb, the lines from 0 to 15 of the vernier would coincide successively with lines

of the limb, and thus any angle may be read accurately to minutes.

The vernier in the figure reads 48′ to the left. A vernier graduated decimally is much more convenient on railway locations than those with the common graduation to minutes. This is principally on account of its adaptedness to running in curves when the 100 feet chain is used. The work can be done with more ease and rapidity. One objection to it is that the tables in general use are calculated for degrees and minutes.

TO ADJUST THE TRANSIT.

Place the instrument firmly at *a*, level it, clamp all fast, and with tangent-screw set the cross-hairs on the point *b*, at any convenient distance. Reverse the telescope on its axis, and fix another point in the opposite direction,

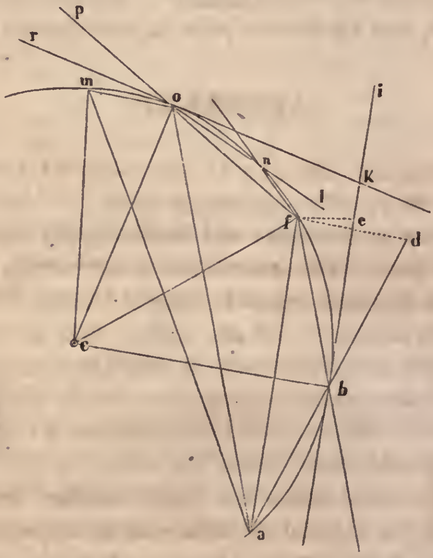
as nearly as possible equidistant from *a*. Now loose the lower clamp and revolve the entire upper part of the instrument half way round on its axis. Clamp fast, and having brought the cross-hairs again to coincide with *b*, reverse the telescope. If the sight strikes as before, the instrument is in adjustment. If not, place another point, *d*, where it does strike, and suppose *c* to be the point previously fixed: the point *e*, midway between *d* and *c*, is then in the straight line. With the adjusting pin carefully place the vertical cross-hair upon *f*, distant from *d* one-quarter of the space *d c*—with tangent-screw set it on *e*, and reverse the telescope. If the points have been correctly placed, and the hair properly moved, the sight will strike *b*, and the adjustment is complete.

After finishing this adjustment, the telescope may still not revolve truly in the meridian. This inaccuracy there is no method of removing in the field. It should be sent to an instrument-maker for repairs.

ARTICLE II.

PRELIMINARY PROPOSITIONS.

1. *In any circle* the angle $o\,c\,f$ at the centre, subtended by the chord $o\,f$, is double the angle $o\,a\,f$, at any part of the circumference on the same side of the chord.



2. *The angle $f\,b\,e$, formed* by any chord $f\,b$, with a tangent at either extremity, is called a *tangential* angle.

and is equal to half the angle $f\ c\ b$ at the centre, or is equal to the angle $f\ a\ b$ at the circumference.

3. *The exterior angle* $d\ b\ f$, formed at the circumference by the two equal chords $a\ b$, $b\ f$, is called a *deflexion* angle, and is equal to the central angle $b\ c\ f$, or double the tangential angle $e\ b\ f$. $d\ f$ is called the *deflexion distance,* and $e\ f$ the *tangential distance.*

4. *The exterior angle* $p\ o\ m$ of two unequal chords, is equal to the sum of their tangential angles, or half the sum of their central angles.

5. *The exterior angle* $i\ k\ o$, formed by tangents, is equal to the central angle $b\ c\ o$, subtended by the chord which connects their points of contact with the curve.

ARTICLE III.

TO AVOID AN OBSTACLE IN THE LINE OF TANGENT.

A GLANCE at the figure will show, that having deflected to c, and placed the instrument at that point, the angle $h\ c\ d$ must be made equal to twice $d\ b\ c$, and the distance

$c\ d$ equal to the distance $b\ c$. Still another deflection at d, equal to the original angle turned, is necessary in order to sight again along the tangent.

Should the obstruction be continuous a parallel line may be run, as from c to f, by deflecting at c an angle

equal to that at *b*, and at *f*, repeating the deflection in order to strike tangent.

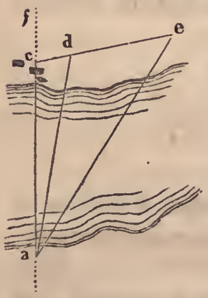
If the angle *d b c* exceeds 4°, and the distance *b c* is greater than 200 feet,—or even with an angle of 2½°, should the distance be greater than 300 feet,—*b c* will differ sensibly in length from *b k*, and a calculation of the latter becomes necessary. To effect this, multiply the natural cosine of the angle *k b c* by *b c*. This result *doubled* will give *b k d*, the length proper along tangent.

Thus:—Suppose *k b c*, the angle deflected, to have been 5°, and the distance *b c* 340 feet. Then ·9962, the natural cosine of 5°, multiplied by 340, gives 338·7 for the distance *b k*. Double this makes *b k d* = 677·4, and shows a difference of 2·6 feet between *b d* and *b c d*.

ARTICLE IV.

SHOULD THE OBSTRUCTION LIE ON THE OPPOSITE BANK OF A STREAM,

AND it is desirable on any account not to run the line from *d*, corresponding to *a d*, set the instrument at *a*, in tangent, and deflect clear of the obstacle to *d*. *Point* at *d*, deflect to *e*, and point also there—marking the angles *c a d*, *d a e*. Chain the base *d e*, and placing the transit at *e*, measure the angle *d e a*. Data are thus obtained sufficient for the calculation of the line *d a*. The object now is to find the point *c* and the angle *d c a*.



The angle *a d e* subtracted from 180° will supply the

angle $c\,d\,a$, so that in the smaller triangle we have obtained two angles and their included side. The distance $c\,d$, and angle $d\,c\,a$ readily follow. The transit standing at e, c is placed, of course, in the prolongation of the base $d\,e$, and the distance $c\,d$ is carefully set off with the rod. Moving the instrument to c, and turning the angle $e\,c\,f = 180° - d\,c\,a$, we are again in tangent.

Example.—Let $c\,a\,d = 6°$, $d\,a\,e = 35°$, $d\,e\,a = 42°$, and the base $d\,e = 200$ feet. Then in the triangle $d\,e\,a$ we have

Nat. sine $d\ a\ e = (\cdot 5736)$: nat. sine $d\ e\ a = (\cdot 6691)$ :: $d\ e = (200) : d\ a$,

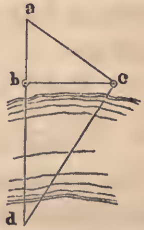
$$\text{Wherefore,} \quad d\ a = \frac{\cdot 6691 \times 200}{\cdot 5736} = 233\cdot 3 \text{ feet.}$$

Again, the angle $c\,d\,a = 77°$. The angle $d\ a\ c$ being $= 6°$, $a\,c\,d$ is consequently $= 97°$, and in the small triangle we have, Nat. sin. $a\ c\ d = (\cdot 9925)$: nat. sin. $c\ a\ d = (\cdot 1045)$ :: $d\ a = (233\cdot 3) : c\ d$.

$$\text{Therefore } c\ d = \frac{\cdot 1045 \times 233\cdot 3}{\cdot 9925} = 24\cdot 564 \text{ feet, and } d\ c\ f$$

$180° - 97° = 83°$.

NOTE.—A common and convenient plan for triangulating a creek is as per figure. Set the instrument at b, fix a point d on the opposite shore, and making $d\ b\ c$ a right angle, place c at any convenient distance. Now move to c, sight to d, and making $d\ c\ a$ a right angle also, fix a, in the same line with b and d. a, c, and d are points in the circumference of a circle whose diameter is $a\,d$, $a\ b : b\ c :: b\ c : b\ d$, and therefore $b\,d = \frac{bc^2}{ab.}$



ARTICLE V.

HAVING GIVEN THE ANGLE $e\,d\,b$, FORMED BY THE INTERSECTION OF TWO STRAIGHT LINES, IT IS REQUIRED TO FIND THE POINT a OR b, AT WHICH TO COMMENCE A CURVE OF GIVEN RADIUS.

Draw the bisecting line $d\,c$. Then the angle $d\,c\,a =$ half the angle $a\,c\,b$ or its equal $e\,d\,b$; and in the triangle $d\,c\,a$, the angle $d\,a\,c$ being a right angle, we have

Rad. of 1 : Nat. Tang. $d\,c\,a$: Rad. $c\,a$: $a\,d$. Therefore $a\,d =$ Nat. Tang. $d\,c\,a \times$ Rad. $c\,a$.

Example 1.—Let $e\,d\,b = 48°$ and $a\,c = 1460$ feet. Here half the angle $e\,d\,b$ or $a\,c\,b = 24°$, the Nat. Tang. of which is ·4452; and multiplying by Rad. 1460, we have 650 feet for the length of $a\,d$ or $d\,b$, the tangents.

2.—*If $a\,d$ be given and radius required,—*

$$\text{Rad.} = \frac{a\,d}{\text{Nat. Tang. } a\,c\,d} = \frac{650}{\cdot 4452} = 1460.$$

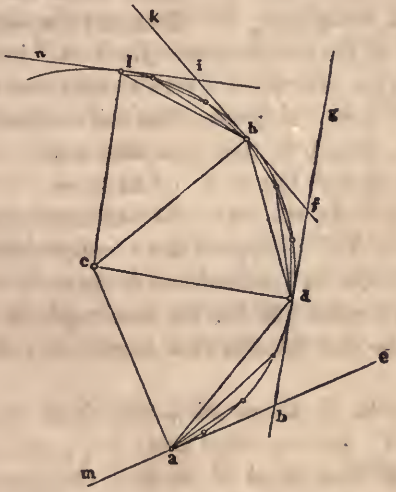
The following rules are approximate, and sufficiently correct for all purposes of location.

To find the degree of curvature of $a\,b$ divide 5730 by the radius in feet; and to find the length of the curve in feet divide the angle $a\,c\,b$ (after reducing minutes to hundredths) by the degree of curvature—the chord in each case being 100 feet in length.

ARTICLE VI.

TO TRACE A CURVE WITH TRANSIT AND CHAIN.

THE degree of curvature and the angle to be turned are known. If the latter is expressed in degrees and minutes, reduce the minutes to hundredths, since the 100 feet chain is used, and divide the whole angle by the degree of curvature. The quotient will be the length of the curve in feet, and the P. T. is at once ascertained.



Let *m a* be the tangent, and *a* the P. C. Place the transit at *a*, index reading 0, and direct the sight along the tangent *m a e*. The first deflection will be half the central angle subtended by the chord used, and all the stakes put in from *a* will be fixed by similar tangential deflections. (*Prelim. Prop.* 1.)

When the point d is reached, the angle $d\,a\,b$, shown on the index, will be half the angle $d\,b\,e$, or its equal $a\,c\,d$, at the centre. Move the instrument to d, sight back to a, and turn to *double the index angle.* The telescope is now directed along the tangent $b\,d\,g$, and the angle $d\,b\,e = a\,c\,d = d\,a\,b + a\,d\,b$, reads on the index. Note this angle in the column of tangents opposite station d. Continue the curve from this new position, precisely as was done at a, and set the point h. Move to h, see that the vernier has not been disturbed, and sight back to d. The index now shows the angle $(d\,b\,e + h\,d\,g)$, and the object is to turn the angle $d\,h\,f$, i. e. *repeat* the angle $f\,d\,h$, as was done before at d, and, at the same time, have the whole angle $(d\,b\,e + h\,f\,g)$ indicated on the plate. To effect this, merely add this angle $f\,h\,d$ to the present reading. It will be found simpler, in practice, to double the entire angle thus far turned, and subtract from the product the last tangent, viz. $d\,b\,e$. The vernier, turned to this resultant angle, will put the telescope in tangent line to h. *And so on.*

Example.—"At sta. 24 + 50 commence a 4° curve to the left for 35° 12′." Suppose this a required duty. First, reducing minutes to hundredths, we have 35°·20, which, divided by 4°, gives 880 feet for the length of the curve. Adding 880 to 24 + 50 it is at once seen that sta. 33 + 30 is the P. T.

Let a be the P. C., = 24 + 50. Now the *deflexion angle* being 4°, the *tangential* angle is 2°, with a chord of 100 feet. With a chord of 50 feet, therefore, the tangential angle is 1°, and this deflexion from tangent $m\,a\,e$ fixes station 25. A deflexion from this latter point of 2°, the chord being 100 feet, fixes station 26. *And so on.*

When you have fixed the point d, = sta. 28, the index reads 7°. Move up to station 28, sight back to the P. C., and turn the index to 14°. This throws you on tangent

Proceed as before, with the 2° deflexions, to sta. 31, $= h$. Move up, and sight back to sta. 28. The index now reads 20°. Multiplying by 2, and subtracting the last tangent, we have the reading of the tangent at $h = 26°$: we have turned 26° of the curve. Continue as before. After putting in sta. 33, to find the deflexion which shall fix the P. T., 33 + 30, say, as 100 feet : 30 feet :: 2° : the required deflexion, = 36′. We may here remark the great convenience of an instrument graduated to *hundredths* of a degree instead of *sixtieths*. In the present example it would be seen immediately that the tangential angle for 100 feet being 2°, for 1 foot it would be 2 *hundredths* of a degree, and for 30 feet it would be 60 hundredths.

Well! when the P. T., = 33 + 30, is fixed, the index reads 30° 36′. Move up, see that the vernier has not been disturbed, and sight back to sta. 31. Now twice the index reading, minus the last tangent, = 61° 12′ — 26°, = 35° 12′, the present tangent, which is the final tangent, which finishes the curve.

The advantage of this manner of running a curve is that the instrument shows at a glance the work done, and therefore errors may be detected with greater facility. By comparing at the P. T. the total index angle with the distance run, the work is tested at once.

The above is recorded in the field book as follows :—

Station.	Distance.	Deflex'n.	Index.	Tangent.	Course.	Mag. Course.	Remarks.
23₀	100ft.				N. 20°·00 W.	N. 20° 05 W.	
24	50						
P. C. +50₀	50						At sta. 24 + 50 commence a 4° curve to the L. for 35° 12′.
25	100	1°·00	1°·00				
26	100	2·00	3·00				
27	100	2·00	5·00				
28₀	100	2·00	7·00	14°·00		N. 34° 07′ W.	
29	100	2·00	16·00				
30	100	2·00	18·00				
31₀	100	2·00	20·00	26·00			
32	100	2·00	28·00				
33	30	2·00	30·00				
P. T. +30₀	70	0·36′	30·36′	35·12′	N. 55°·12 W.	N. 55° 18′ W.	
34	100						

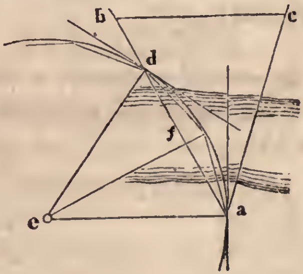
In running compound and reversed curves the operation is quite as simple as the foregoing. A point is fixed at the P. C. C., or P. R. C., and turning into tangent at that point, the second curve is traced from this *tangent*, without regard to what precedes. In reverse curving, it is a good plan to adjust the index in such manner at the P. R. C., that when we turn into tangent it will read 0.

This saves troublesome work, and it is advisable moreover to show in the field-book the contained angle *of each curve*, as well as the test of the two tangents with the magnetic course.

ARTICLE VII.

TO TRIANGULATE ON A CURVE.

SET the transit at *a*, and, as usual, sight back, and turn into tangent. Estimate the distance to the farther bank—do it liberally—and make a deflexion around the curve, corresponding to your estimated distance. Fix a point *b* in this line. Measure any convenient angle, *b a c*, and set



the point *c*. Move to *b*, measure the base *b c*, the angle *a b c*, and, before lifting the instrument calculate the line *b a*. If the angle turned from tangent to *d* exceeds 4°, and the distance is greater than 200 feet, the chord *a d* must also be calculated, as per example, and the difference between this and *b a* will be the distance *b d* to the point *d*, in the curve, which can be fixed from *b*.

Should b fall between d and a the operation is analogous.

Example.—Let a be a point in a 6° curve. Having set the transit, and turned into tangent, the distance to the farther verge is estimated 400 feet. The tangential angle for 100 feet is 3°, and to fix d, 400 feet distant, is consequently 12°. Deflect this angle, fix a point in line, and complete the triangulation, as previously illustrated in Art. IV., p. 22. Suppose $a\,b$ found equal to 472 feet. Now the tangential angle to d = half the central angle, $= f\,e\,a$, = 12°; and to find the length of the chord $a\,d$, we have, in the triangle $e\,f\,a$,

Rad. : Sin. $a\,e\,f$ :: $e\,a$: $a\,f$, that is

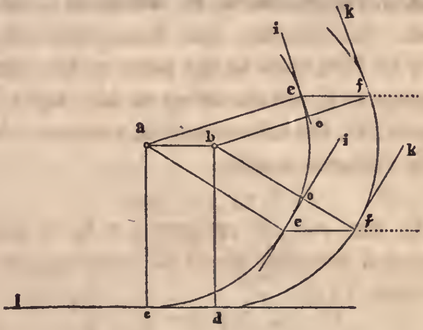
Rad. of 1 : Nat. Sin. 12° :: 955·4, the Rad. of the 6° curve : half the chord required. Wherefore $a\,d$ = twice the Nat. Sin. 12° × 955·4 = ·2079 × 2 × 955·4 = 397·25 feet. Subtracting this from 472, we have the distance, 74·75 feet, back to the point in the curve. Move the instrument to d, set the index at 12°, sight back to a, and turning to 24°, the telescope is in tangent. A deflexion of 3° will fix the next station.

Note.—In this case, if preferred, a third proportional might be formed with the chord of crossing, as shown in the note to Art. IV.

ARTICLE VIII.

TO CHANGE THE ORIGIN OF ANY CURVE, SO THAT IT SHALL TERMINATE IN A TANGENT PARALLEL TO A GIVEN TANGENT.

LET *d f* be the located curve, terminating in a tangent *f k*, and the nature of the ground requires that it should terminate in the tangent *e i*, parallel to *f k*. At *f*, the telescope being directed along the tangent *f k*, turn to the

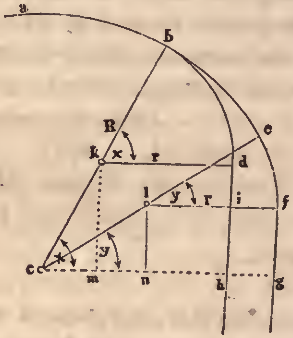


right an angle equal to the central angle *d b f*, previously turned to the left on the curve. This will direct the telescope along *e f*, parallel to *d l*. Measure *e f*, and go back on the tangent, *d l*, a distance, *c d*, equal to it. The curve, retraced from *c* and consuming the same angle, will terminate tangentially in *e i*. An example in this case is not necessary.

ARTICLE IX.

TO CHANGE A P. C. C. SO THAT THE SECOND CURVE SHALL TERMINATE IN A TANGENT PARALLEL TO A GIVEN TANGENT.

LET $a\,b\,d$ be the compound curve, located and terminating in the tangent $d\,h$. Continue the larger curve to e, and from e, with radius $e\,l = k\,b$, describe the curve $e\,f$, terminating tangentially in $f\,g$, parallel to $d\,h$. From c, the centre of the larger curve, let fall upon $f\,g$ the perpen-



dicular $c\,g$, and fill up the figure as above. Call the radii respectively R and r, the angle $b\,k\,d$, or its equal, $k\,c\,m$, x, and the angle $e\,l\,f$, or its equal, $l\,c\,n$, y. Let the distance $i\,f$, or $h\,g$, be named D. Now the line $c\,g$ is made up of the lines $c\,m + m\,h + h\,g$, i. e., $c\,g =$ cosin. $x\,(\mathrm{R} - r) + r + \mathrm{D}$. $c\,g$ is also made up of the lines $c\,n + n\,g$, i. e., $c\,g =$ cosin. $y\,(\mathrm{R} - r) + r$. Therefore cosin. $x\,(\mathrm{R} - r) + r + \mathrm{D} =$ cosin. $y\,(\mathrm{R} - r) + r$, and reducing,

$\text{cosin. } y = \dfrac{\text{cosin. } x\,(R - r) + D}{(R - r)}$; so that the distance if, or hg, measured rectangularly between the two tangents, being added to the nat. cosin. x, will give the nat. cosin. of the angle elf, to be turned on the smaller curve. The angle y, subtracted from the angle x, gives of course the angle bce, to be advanced on the larger curve; or, dividing this angle by the degree of curvature of ab, we find the distance from b to e the P. C. C. proper.

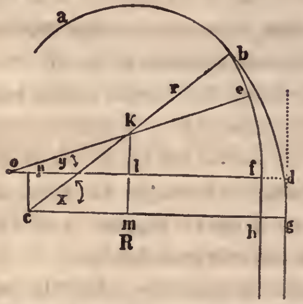
If ef be the second curve located, and the tangent to be touched lies *within*, it is evident that we must *retreat* upon the large curve, and, by *subtracting* D from the cosine of the angle y, we obtain the cosine of the angle x.

Example.—Suppose ab a 3° curve located, and compounding, at b, into a 6° curve, which latter is continued to the right through an angle of 42°. At the P. T. we discover that the proper tangent is 64 feet to the left. We must throw our curve *out*, then—we must *advance* on the 3° curve a certain distance. How to find this distance: The radius of a 3° curve = 1910; the radius of a 6° curve = 955·4; $R - r$, therefore, = 954·6. The nat. cosin. 42° = ·7431. Now, by the formula just obtained, we have $\dfrac{\text{cos. } x\,(R - r) + D}{(R - r)} = \dfrac{(\cdot 7431 \times 954 \cdot 6) + 64}{954 \cdot 6} =$ ·8101 = nat. cosin. 35° 53′. Subtracting this from 42°, we have 6° 07′, the angle to be advanced on the 3° curve; or, reducing minutes to hundredths, and dividing by 3°, we find 204 feet, the distance from b to the correct P. C. C.

ARTICLE X.

SHOULD THE SECOND CURVE BE ONE OF LONGER RADIUS THAN THE FIRST,

Our illustration takes simpler form, and the application of D varies *vicê versa.*



See figure, analogous to that of the previous problem. Here $o\,f$, $c\,g$ are equal and parallel radii; $f\,h$ a perpendicular connecting them. Draw its fellow, $c\,n$. Then, $n\,f = c\,h$, and, consequently, $h\,g = o\,n$. Again, letting $k\,m$ fall, perpendicular to $c\,g$, we have $c\,m = n\,l$, and $o\,l = c\,m + o\,n$; i. e., cosin. y (R — r) = cosin. x (R — r) + D.

We observe that, with a curve of this nature, in order to throw the line *farther out*, it is necessary to *go back*, toward b; or, having located to g, if the object tangent lie *within*, we must *advance* toward e.

Example.—Suppose $a\,b$ a 5° curve, b the P. C. C., and $b\,g$ a 2° curve. Setting the instrument at g, the P. T., and turning into tangent, we find that we are a distance $h\,g$, = 53 feet, too far to the *left.* The first question is, what angle have we turned on the second curve. Let it

be 28°. Now we know, that, in order to strike farther to the *right*, we must advance on the 5° curve. Consequently, D must be *added* to the cosine of 28°, to give us the cosine of the proper angle for the 2° curve; and the difference between 28° and this newly found angle will be the angle we are to advance on the 5° curve. Thus:—the rad. of a 5° curve = 1146 feet, that of a 2° curve = 2865 feet, and their difference = 1719 feet. The nat. cos. of 28° = ·8829. Then $\frac{(\cdot 8829 \times 1719) + 53}{1719} = \cdot 9137$, = nat. cos. 23° 58'. This, subtracted from 28°, leaves 4° 02', = 80 feet, from *b* to the correct P. C. C.

Synopsis of the preceding formulæ.

Call D the distance between tangents as before, *a* the angle of the second curve located, and *b* the angle of the same curve to be substituted for it.

FIRST, when the second curve has the *smaller* radius—

Tangent falling *within* the point, cosine $b = \frac{\cos. a\,(R - r) + D}{(R - r)}$.

Tangent falling *without* the point, cosine $b = \frac{\cos. a\,(R - r) - D}{(R - r)}$.

SECOND, when the second curve has the *larger* radius—

Tangent falling *within* the point, cosine $b = \frac{\cos. a\,(R - r) - D}{(R - r)}$.

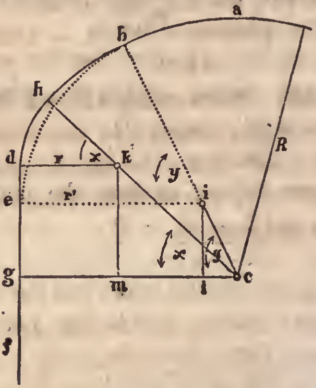
Tangent falling *without* the point, cosine $b = \frac{\cos. a\,(R - r) + D}{(R - r)}$.

Very little attention will familiarize these formulæ, and render the field practice easy.

ARTICLE XI.

HAVING LOCATED THE COMPOUND CURVE $a\,b\,d$, TERMINATING IN THE TANGENT $d\,f$, IT IS REQUIRED TO FIND THE P. C. C. b, AT WHICH TO COMMENCE ANOTHER CURVE OF GIVEN RADIUS, WHICH SHALL ALSO TERMINATE TANGENTIALLY IN $d\,f$.

PLOT the curves as per figure. From c let fall $c\,g$, perpendicular to the tangent, $d\,f$. From k and i, the lesser centres, drop $k\,m$, $i\,l$, perpendicular to $c\,g$. Call the great radius R, the smaller radius r, and the *intended* radius of



the second curve r'. Likewise name $h\,k\,d$, the angle of the small curve located, x, and $b\,i\,e$ the angle to be found for the *proposed* curve, y. Now, the tangent $d\,f$, and the curve $a\,b$, lying unstirred, the line $c\,g$ is an unvarying distance, and it is made up of the lines $c\,m + m\,g$, i. e., $c\,g = (\mathrm{R} - r)$ cosin. $x + r$. It also consists of the lines $c\,l$

$+ lg$, i. e., $cg = (R - r') \text{ cosin. } y + r'$, and, reducing, nat. cosin. $y = \frac{(R - r) \text{ cosin. } x + r - r'}{(R - r')}$. This, therefore, is the formula by means of which we can ascertain the point b, as follows:—

Example.—Imagine a 2° curve, $a\,h$, compounding into a 6° curve, $h\,d$, which terminates at d, in the tangent $d\,f$. The tangent lies well; the curve $a\,h$ likewise; but it is desired to throw the line to the left, on better ground, between d and h, by means of an intercalary 4° curve. We wish, then, to know the distance, $h\,b$, back to the new P. C. C.

The radius of a 2° curve = 2865 feet, of a 6° curve = 955·4 feet, and their difference $(R - r) = 1909{\cdot}6$. The radius of a 4° curve = 1433, and the difference $(R - r')$ is, therefore, 1432 feet. Let $h\,k\,d$, the angle turned on the 6° curve, be 41°, the nat. cos. of which = ·7547.

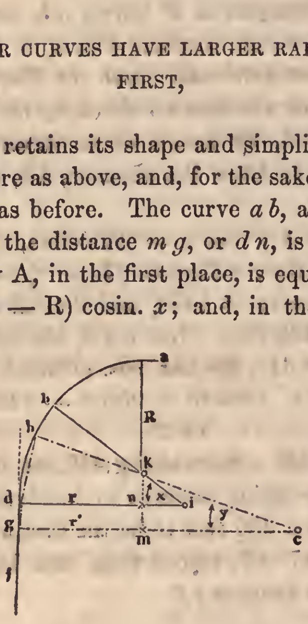
Then, $\frac{(\cdot 7547 \times 1909{\cdot}6) + 955{\cdot}4 - 1433}{1432} = \cdot 6728$, = nat. cosin. 47° 43′. Subtracting 41°, we have 6° 43′, the angle $h\,c\,b$. Reducing minutes to hundredths, and dividing by 2°, we find 336 feet to be the distance from h to b. A 4° curve of 47° 43′, traced from this latter point, will terminate in the tangent $e\,f$.

ARTICLE XII.

IF THE LATTER CURVES HAVE LARGER RADII THAN THE FIRST,

THE solution retains its shape and simplicity.

Draw the figure as above, and, for the sake of uniformity, name the radii as before. The curve $a\,b$, and tangent $d\,f$, being constant, the distance $m\,g$, or $d\,n$, is here constant. Call it A. Now A, in the first place, is equal to $d\,i - n\,i$, i. e., $= r - (r - \text{R})$ cosin. x; and, in the second place



it is equal to $g\,c - m\,c$, $= r' - (r' - \text{R})$ cosin. y; wherefore $r - (r - \text{R})$ cosin. $x = r' - (r' - \text{R})$ cosin. y, and consequently cosin. $y = \dfrac{(r - \text{R}) \text{ cosin. } x + r' - r}{(r' - \text{R})}$.

Example.—Suppose $a\,b$ a 7° curve, compounding, at b, into a 5° curve, $b\,d$, which latter subtends an angle of 38°, and terminates in the tangent $d\,f$. We wish to substitute a terminal 2° curve, $h\,g$, and to know the position, h, of the new P. C. C.

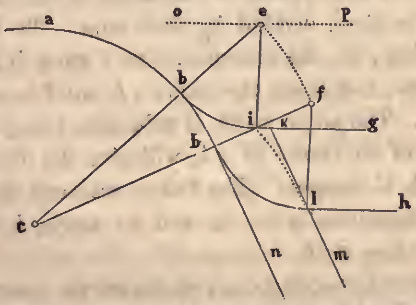
The radius of a 7° curve = 819 feet = R. r and r',

the radii respectively of 5° and 2° curves, are equal to 1146, and 2865 feet. $r - R$, therefore, $= 327$, and $r' - R = 2046$ feet. The nat. cosin. of $38° = \cdot788$. Then, by the formula, $\frac{(\cdot788 \times 327) + 2865 - 1146}{2046} =$ $\cdot9661$, $=$ the nat. cosin. of 14° 57′, the angle to be turned on the 2° curve. Subtracting this from 38°, we have 23° 03′, the angle to be continued on the 7° curve. Reducing minutes to hundredths, and dividing by the degree of curvature, 7°, we find 329 feet, the distance from *b* to the new P. C. C. *h*.

ARTICLE XIII.

TO CHANGE A P. R. C., SO THAT THE SECOND CURVE SHALL TERMINATE IN A TANGENT PARALLEL TO A GIVEN TANGENT.

Let *a d l* be the reverse curve, located and terminating



in the tangent *l h*. Call the radius *c b*, R, and the radius *b e*, *r*. Suppose *i g* the given tangent. At a distance

from it equal to $i\,e$, the radius of the second curve, draw the parallel line, $o\,p$. With c as a centre, and radius $c\,f$, $= R + r$, describe the integral curve, $f\,e$, cutting $o\,p$ in e. e, then, is the centre of the curve adjusted.

Application.

Place the transit at the P. T., l, and turn into a tangent, $l\,m$, parallel to $d\,n$, the common tangent of the two curves at d. Unless some wide mistake has been made, the distance $l\,k$, measured along this line to $i\,g$, the tangent proper, will be about equal to the distance $e\,f$, and we shall have the proportion, $c\,f : f\,e :: c\,d : d\,b$, i. e., $R + r : e\,f :: R : d\,b$, which gives $\frac{f\,e \times R}{R + r}$, as a simple formula for finding the distance back from d to b, the correct P. R. C. This rule, though sufficiently true for most cases, is not mathematically justifiable. It will be seen that $e\,f$, or its equal $i\,l$, the distance we wish to measure, is a *curving* distance, part of the circumference of a circle concentric with $a\,b$. Its radius is $(R + r)$, therefore its degree of curvature $= \frac{5730}{(R + r)}$, or, more simply, equals the *product* of the degrees of curvature of the curves composing the reverse, divided by their *sum*. To be strictly accurate, then, set the instrument at l, turn into tangent $l\,m$ as before, and trace the curve $i\,l$, until it strikes the tangent $i\,g$. The angle which $i\,l$ subtends, being divided by the degree of curvature of $a\,b$, will give the distance, $d\,b$, to the P. R. C. proper. The curve retraced from b, will terminate tangentially in $i\,g$, and its angle, $b\,e\,i$, will be equal to $d\,f\,l - d\,c\,b$.

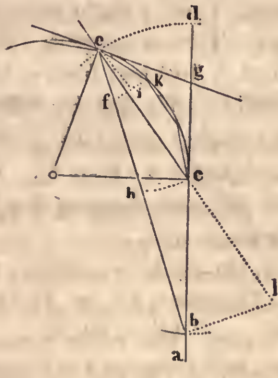
Example.—Let $a\,d\,l$ be a reverse curve, composed of a 3° curve, $a\,d$, and a 6° curve, $d\,l$. Let the angle $d\,f\,l$ be equal to 52°, and suppose the distance $l\,f$ to have been

found 34 feet. Being part of a 2° curve, it therefore subtends a central angle of 41′. This corresponds to a distance of 23 feet, to be gone back on the 3° curve, and 52°·00 — 41′ = 51° 19′, the angle to be turned from *h* on the 6° curve, in order to strike the tangent *i g*.

ARTICLE XIV.

HOW TO PROCEED WHEN THE P. C. IS INACCESSIBLE.

In the figure, drawn to illustrate this case, let *c* be the point of curvature, *c a* the tangent, and *c k e* the curve. Now the angle *d c e*, included between the tangent and



any chord, as *c e*, fixing the point *e*, is known. Make *c b* along tangent, equal to *c e*, and connect *b e*. If a circle were now described from *c* as a centre, with radius *c e* or *c b*, *d*, *e*, and *b*, would be points in its circumference, and

the angle $d\,b\,e$ at once proven equal to half the angle $d\,c\,e$. With proof precisely similar, $d\,c\,e =$ half of $d\,g\,e$, and, consequently, $d\,b\,e$ is equal to one-fourth of the central angle subtended by the chord $c\,e$.

Example.—Suppose c to be the inaccessible point of curvature of a 6° curve, $c\,k\,e$. It is concluded to run to the third station, e. First we must calculate the length of the chord $c\,e$. The angle $d\,c\,e = 9°$, and from Art. VII. we have

Rad. of 1 : 955·4 : : nat. sin. 9° $=$ ·1564 : $\frac{e\,c}{2}$, whereby $e\,c$ is shown equal to 298·8. Place the transit then at b, 298·8 feet distant from the P. C., and deflect to the left an angle of 4° 30′, equal to half the angle $d\,c\,e$. This is in line to e, and $b\,e$ must likewise be calculated as follows:

In the triangle $b\,c\,h$ we have

Rad. of 1 : nat. cosin. 4° 30′ $=$ ·9969 : : $b\,c = 298{\cdot}8 : b\,h = \frac{b\,e}{2}$, whereby $b\,e$ is shown equal to 595·7 feet. Arriving at e, the index reads 4° 30′. Sight back to b, turn to 18°, and the telescope will be in tangent. Suppose, however, that having reached f, 100 feet from e, this latter point is also found inaccessible. We find k a different point in the curve, thus:—The angle $f\,e\,g = 18° - 4°\,30' = 13°\,30'$, and the tangential angle $g\,e\,k = 3°$. Consequently the angle $f\,e\,k = 10°\,30'$, and, drawing the bisecting line $e\,i$, we have, in the triangle $e\,f\,i$,

Rad. of 1 : nat. sin. 5° 15′ $=$ ·0915 : : $e\,f = 100 : f\,i = 9{\cdot}159$ feet. Therefore $f\,k = 18{\cdot}318$ feet, and the angle $e\,f\,k = 90° - 5°\,15' = 84°\,45'$. At f deflect this angle to the right, and measure the distance $f\,k$ carefully with the rod. At k, sighting back to f, and turning the equal angle $f\,k\,e$, the telescope will be directed to e, and the curve may be continued.

If it is inconvenient to run the line $b\,e$, the point e may

be reached thus:—Fix the P. C. Find the tangential distance *d e*, corresponding to the angle *d c e*. Carefully with the rod lay off *b l*, equal to it, at right angles to *b e* Set the transit at *l*, and, in line with *c*, put in *e*.

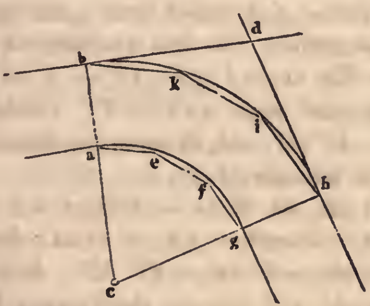
The distance *b l* should not exceed 10 or 12 feet.

NOTE.—The foregoing illustrations will apply when the P. T. is likewise inaccessible.

ARTICLE XV.

TO AVOID OBSTACLES IN THE LINE OF CURVE.

LET *b k h* be the curve. We can either follow the tangents *h d*, *d b*, or trace a parallel curve, *g a*, *within* the first; which tracing is effected thus:—Set the instrument



at *h*, the P. C., and offset any distance *h g*, at right angles to the tangent *h d*. It will be observed that as the distance *h g increases*, the distance *g a decreases*, whilst the angle subtended by *g a* remains equal to that subtended

by $h\,b$; i. e., our *deflexions* on the offset curve stand unchanged, but the corresponding *chords*, $g\,f$, $f\,e$, &c., are *less* than their equivalents, $h\,i$, $i\,k$, &c., along $h\,b$. To find their length, $h\,i$, $i\,k$, &c., being equal to 100 feet, we have the proportion, $c\,h : c\,g :: h\,i : x$; i. e., R : rad. — $h\,g$: : 100 feet : x, where x symbols the unknown chord. Now set the transit at g, turn into tangent parallel to $h\,d$, and with the shortened chord, fix $f\,e\,a$. Rectangularly to the tangents at these points, and distant $h\,g$, will be i, k, b of the curve proper.

Example.—Let $b\,h$ be a 4° curve, and the offset distance 85 feet. The radius then is 1433, and

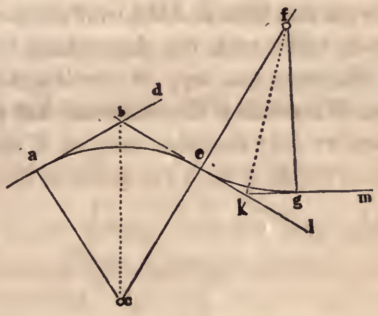
1433 : 1348 : : 100 : 94, the short chord.

To follow the tangents, suppose the angle $b\,c\,h = 42°$. Then by Art. V. we find the tangent $h\,d = 550$ feet, which distance we duly measure, and, at d, deflecting 42°, lay off an equal distance to b, the point of tangency.

ARTICLE XVI.

HAVING GIVEN THE ANGLES $d\,b\,k$, $m\,k\,l$, AND THE DISTANCE $b\,k$, IT IS REQUIRED TO FIND THE RADII $c\,e$, $e\,f$ OF THE EASIEST REVERSE CURVE WHICH SHALL UNITE $a\,d$, $k\,m$.

THE angle $d\,b\,e$ is equal to the angle $a\,c\,e$, half of which is $b\,c\,e$. So likewise $e\,f\,k$ is equal to half of $l\,k\,m$.



Then, [nat. tang. $b\,c\,e$ + nat. tang. $e\,f\,k$] : nat. tang. $b\,c\,e$: : $b\,k$: $b\,e$, and $b\,k - b\,e = e\,k$. Wherefore rad. $c\,e$ $= \dfrac{b\,e}{\text{nat. tang. } b\,c\,e}$ and rad. $e\,f = \dfrac{e\,k}{\text{nat. tang. } k\,f\,e}$.

Example.—Suppose the angle $d\,b\,e = 54°\ 30'$, the angle $l\,k\,g = 33°\ 20'$, and the distance $b\,k = 832$ feet.

Therefore the angle $b\,c\,e = 27°\ 15'$, the nat. tang. of which is ·5150, and the angle $e\,f\,k = 16°\ 40'$, the nat. tang. of which is ·2994. The sum of the tangents = ·8144. Then, to find $b\,e$, we have

As ·8144 : ·5150 : : 832 : 526, and subtracting this from $b\,k$, we have $e\,k = 306$ feet.

Again, the radius $c\,e = \dfrac{526}{\cdot 515}$, and the radius $e\,f = \dfrac{306}{\cdot 2994}$, = 1022 feet.